$\beta - P$ Conectedness in L-Bitopological spaces

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***Abstract*— This paper introduce open sets and $p-$closed sets into L-Bitopological spaces, and based on this we introduce some related definitions and theorems about $\beta - P$ closed set and $\beta - P$ open sets . Furthermore, we give a new concept about $\beta - P$ local-connectivity .Then, it point out $\beta - P$ local-connectivity has two properties of topological invariance and finitely productive property and proves the other relevant theories.**

Index Terms— L- **bitopological spaces, $\beta - p$ local-connectivity, topological invariance, finitely productive property**

I. Introduction

Since J. C. Kelly introduced the concept of double topological space, many scholars has a great interest about researching dual topology, then introduced L-bitopological spaces on the basis of L-topological space, and makes a long-term study of the separation .Connectivity is an important branch of fuzzy topology, the domestic scholars have studied the variety of connectivity, such as $\theta-$connectivity[3], connectivity[4], stratified connectedness [5], disconnectedness branches e .c. Due to the concept of connectivity is closely related to geometric closure , many connectivity is also defined based on the definition of different closure concepts. In this paper,we introduced $\beta - P$ open sets and $\beta - P$ closed sets in L-Bitopological spaces on the basis of $\beta - P$ open sets and $\beta - P$ closed sets .In this case ,we defined a new connectivity in L- bi-topological spaces which is called $\beta - P$ local-connectivity ,and the study of the connectivity has gotten some good properties.

II. Preliminary knowledge

A. *L-bitopological spaces*

Definition 2.1: Let L be a F lattice ,that is a completely distributive lattice with the reverse involution ,let X be a common set and let L^X is a set that contains the whole $L-fuzzy$ sets on X ,0 and 1 respectively expressed the minimum and maximum in L , $M(L)$ and $M^*(L^X)$ respectively expressed all molecules of L and L^X ,we record $L-bts$ as $L-$bitopological space and $A_{\delta_1^-}$ as the closure of A in (L^X,δ_1)[6].

B. *Final Stage The Related Defines And Conclusion About $p-$open set And $p-$close Set*

Definition 2.2: Let (L^X,δ) be $L-bts$, $A\in L^X$ and record as $p-$ open set ,if and only if have open set U ,makes $A\le U\le A^-$;If A is a $p-$ open set, we call A' is a $p-$ close set .The whole $p-$ open sets in (L^X,δ) are denoted by $LPO(L^{X^*})$,and the whole $p-$ close sets are denoted by $LPC(L^{X^*})$[7].

Note: The close set in $L-bts$ must be $p-$ close set, and on the contrary generally does not set up.

Lemma 2.1: Let (L^X,δ) be $L-bts$, then $\delta\subset LPO(L^{X^*})$, $\delta'\subset LPC(L^{X^*})$.

Definition 2.3: Let (L^X,δ) be $L-bts$, $A\in L^{X^*}, B\in L^{X^*}$, then:

(a)The union of all $p-$ open sets which contained by A is called internal of A 's $LE-p^*$,record as $A^{*\Delta}$,that is

$$A^{*\Delta}=\{B\in LPO(L^{X^*})B\le A\}.$$

(b) The union of all $p-$ close sets which contained by A is called external of A 's $LE-p^*$,record as $A^{*\leftarrow}$,that is

$$A^{*\leftarrow}=\{B\in LPC(L^{X^*})A\le B\}$$

Lemma 2.2: Let (L^X,δ) be $L-bts$, $A,B\in L^{X^*}$,then

(a) $A^\circ\le A^{*\Delta}\le A\le A^{*\leftarrow}\le A^-$.

(b) If $A\in\delta\cap LPC(L^{X^*})$,then $A\in\delta'$.

(c) If $A\in\delta\cap LPO(L^{X^*})$,then $A\in\delta$.

(d) If $A\le B$,then $A^{*\leftarrow}\le B^{*\leftarrow}, A^{*\Delta}\le B^{*\Delta}$.

(e) $(A\vee B)^{*\leftarrow}=A^{*\leftarrow}\vee B^{*\leftarrow}, (A\vee B)^{*\Delta}=A^{*\Delta}\vee B^{*\Delta}$.

(f) The arbitrary intersection of $p-$ closed sets is $p-$ closed set,and the arbitrary intersection of $p-$ open sets is $p-$ open set.

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C. Figures The Introduction of $\beta - p$ open set And $\beta - p$ close set

According to the preparation,we introduce $\beta - p$ open set and $\beta - p$ close set into $L-$bitopological spaces,and give some related properties and theorems[7] about $\beta - p$ open set and $\beta - p$ close set on the basis of $p-$closed set and $p-$open set,and give the corresponding proof.

Definition 3.1: Let (L^X, δ) be $L-bts, \beta \in L, A \in L^X$,record as $l_\alpha(A) = \{x \in X | A(x) > \beta\}$,let $\beta \in L - \{0\}$,if $l_\beta(A^{*\leftarrow}) = l_\beta(A)$, then call A as $\beta - p$ open set;If A' is $\beta - p$ close set,then we call A as $\beta - p$ open set .

Lemma 3.1:Let (L^x, δ) be $L-bts, A \in L^X$,if A is $\beta-$close set,then A is $\beta - p$ close set.

Proof: If A is $\beta-$close set,then about $\beta \in L - \{0\}, A \in L^{X^*}$,when $\gamma \geq \beta$,there is $l_\beta(A^-) = l_\beta(A)$,due to $A^{*\leftarrow} \leq A^{*\leftarrow}$,then there is $l_\beta(A^{*\leftarrow}) = l_\beta(A)$,so A is a $\beta - p$ close set.

Lemma 3.2:Let (L^x, δ) be $L-bts, A \in L^X$,if A is a $\beta-$open set,then A is $\beta - p$ open set.

Lemma 3.3:Let (L^x, δ) be $L-bts, A \in L^X$,If A is a $p-$ close set is the necessary and sufficient condition of A is $1-p$ close set.

Proof :

"$\Leftarrow$" Clearly established.

"$\Rightarrow$" Due to "A" is $1-p$ close set,so $\forall \beta \geq 1' = 0, l_\beta(A^{*\leftarrow}) = l_\beta(A)$

If $A^{*\leftarrow} \neq A$,then there is $x \in X$,cause $A^{*\leftarrow}(x) > A(X)$,marked $\beta_0 = A(x) \geq 0$,then $x \notin l_{\beta_0}(A^{*\leftarrow})$is conflict with (*) ,so $A^{*\leftarrow} = A$,that is A is $\beta - p$ close set.

Lemma 3.4:Let (L^x, δ) be $L-bts, A, B \in L^X, \beta \in L - \{0\}$,then

(a) If $A \leq B$,then $l_\alpha(A) \subseteq l_\alpha(B)$.

(b) $l_\alpha(A \wedge B) = l_\alpha(A) \cup l_\alpha(B)$.

(c) $l_\alpha((A \wedge B)^{*\Delta}) = l_\alpha(A^{*\Delta}) \cup l_\alpha(B^{*\Delta})$.

(d) $l_\alpha((A \vee B)^{*\Delta}) = l_\alpha(A^{*\Delta}) \cap l_\alpha(B^{*\Delta})$.

(e) $l_\alpha(A^{*\leftarrow} \wedge B^{*\leftarrow}) = l_\alpha(A^{*\leftarrow}) \cap l_\alpha(B^{*\leftarrow})$.

(f) $l_\alpha(A^{*\leftarrow} \vee B^{*\leftarrow}) = l_\alpha(A^{*\leftarrow}) \cup l_\alpha(B^{*\leftarrow})$.

D. $\beta - p$ local-connectivity

1.Definitions about $\beta - p$ local-connectivity

Definition 4.1.1:Let (L^X, δ)be $L-bts, A, B \in L^X, \alpha \in L - \{0\}$, if $A^{*\leftarrow} \wedge B \leq \beta'$ and $A \wedge B^{*\leftarrow} \leq \beta'$,then call A and B are $\beta - p$ insular.

Definition 4.1.2:Let (L^X, δ) be $L-bts$, $S \in L^X$,if there is no $A, B \in L^X$,to make A and B are $\beta - p$ insular,and $A \vee B = S$, $A > \beta'$, $B > \beta'$,then call S is Connected set in (L^X, δ).Particularly, when 1_X which is the Maximum element in L^X is $\beta - p$ Connected set;Otherwise,call (L^X, δ) is $\beta - p$ disconnected space.

Definition 4.1.3: Let (L^X, δ) be $L-bts$, $x \in L^X$,if every neighborhood of A contains a $\beta - p$ connected neighborhood V ,then call x is $\beta - p$ local-connectivity;Otherwise,call x is not $\beta - p$ local-connectivity[8].

Definition 4.1.4: If every point of (L^X, δ) is $\beta - p$ local-connectivity,then call (L^X, δ) is $\beta - p$ local-connected space.

2 .Basic properties of $\beta - p$ local-connectivity

Theorem 4.2.1: The local connected space must be $\beta - p$ local-connected space.

Proof : From the definition of local-connected spaces ,we can know that every neighborhood of A contains a $\beta - p$ connected neighborhood V ,the V is open set .By definition 4.2 in the literature [3] ,we can know that the connected open set must be $\beta - p$ open set of V .As for $\forall x \in L^X$,if every neighborhood of A contains a $\beta - p$ connected neighborhood V ,then (L^X, δ) is $\beta - p$ local-connected space according to definition 4.1.4.

On the contrary, does not necessarily set up.

Theorem 4.2.2: Let (L^X, δ) be $L-bts$, then the following conditions are equivalent:

(a) L^X is a $\beta - p$ local-connected space.

(b)Arbitrary $\beta - p$ connected branch of L^X 's arbitrary $\beta - p$ open set is a $\beta - p$ open set.

(c) L^X is a connected base.

Proof :

(a)$\Rightarrow$(b):

Let U is a arbitrary $\beta - p$ open set of $\beta - p$ connected space,c is arbitrary $\beta - p$ connected branch of U ,as for arbitrary $x \in c \in U$,then $U_X \in U$ .And because

L^X is $\beta - p$ local-connected,there is a connected neighborhood V ,and V also is $\beta - p$ connected neighborhood of subspace U [9].

$\Rightarrow$(c) :

Let α is a set family that formed by all $\beta - p$ open set of L^X ,then α is a base of L^X ,also because $\beta - p$ connected branch is $\beta - p$ connected subset,then α is connected base of L^X .

$\Rightarrow$(a) :

Let L^X has a $\beta - p$ connected base α ,then each member of α are all $\beta - p$ connected sets,as for $\forall x \in L^X$,make $\alpha_x = \{A \in \beta | x \in B\}$,then $x \in \alpha_x \subset \alpha$,that α_x is $\beta - p$ connected,we can know L^X is $\beta - p$ local-connected by the definition 4.1.3.

Theorem 4.2.3: Let $(L^X{}_1, \delta_1)$ is a $\beta - p$ local-connected space, (X,T) is a topological space, $f : L^X \to X$ is a continuous open mapping,then we can construct that $(L^X{}_2, \delta_2)$ is a $\beta - p$ local-connected space by (X,T).

Proof :Let L^X is a $\beta - p$ local-connected space, then we can know there is a $\beta - p$ connected base in L^X by the definition 4.2.2.As for f is a continuous open mapping,to make $\beta_0 = \{f(B) | B \in \beta\}$,then $\forall B \in \beta, f(B), \forall B \in \beta, f(B)$ is $\beta - p$ connected open set of X ,so β_0 is $\beta - p$ connected open set family of X .As for arbitrary $\beta - p$ open set of X , f is $\beta - p$ connected and surjection,so:

$$A = f(f^{-}(A)) = f\left(\bigcup_{B \in \beta_1} B\right) = \bigcup_{B \in \beta_1} f(B)$$

Then β_0 is a $\beta - p$ open connected base of X .

By the theorem 4.2.2,we can know $(L^X{}_2, \delta_2)$ can structured by (X,T) is $\beta - p$ connected space .

We call the nature is topological invariance which is can keep the same nature under $\beta - p$ continuous mapping in topological space.

Theorem 4.2.4: If $L^X{}_1$, $L^X{}_2$, ... $L^X{}_n$ are also $\beta - p$ local-connected space,then $L^X = L^X{}_1 \times L^X{}_2 \times .... \times L^X{}_n$ is $\beta - p$ local-connected space too.

Proof :Let $L^X{}_i (i = 1,2,..., n)$ is $\beta - p$ connected space,from theorem 4.2.2, we can know that there is a $\beta - p$ connected base $V_i (i = 1,2,..., n)$ in $L^X{}_i$,then make $V = \{V_1 \times V_2 \times ... \times V_n | V_i \in V(i = 1,2,..., n)\}$,and from literature [10],we can know $\beta - p$ connected nature is finitely productive property ,so V is a $\beta - p$ connected base of product space L^X .Also,from theorem4.2.2,we can know that product space $L^X = L^X{}_1 \times L^X{}_2 \times .... \times L^X{}_n$ is also a $\beta - p$ local-connected space.

Some properties P of topological spaces are called finite integrable properties, if there are $n \geq 1$ arbitrary topological space $L^X{}_1$, $L^X{}_2$, ... $L^X{}_n$ have property P ,and contained product space $L^X = L^X{}_1 \times L^X{}_2 \times .... \times L^X{}_n$ also have property P ,then $\beta - p$ local-connected nature is finitely productive property.

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